CALCULATION OF THE BOUNDARY LAYER ON A DIELECTRIC FLAT PLATE IN A FLOW OF INCOMPRESSIBLE ANISOTROPICALLY-CONDUCTING FLUID, IN THE PRESENCE OF A UNIFORM MAGNETIC FIELD

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Making use of the results in [1-3], the problem of the boundary layer on a flat plate is solved for incompressible flow of an anisotropically-conducting fluid in the presence of a uniform magnetic field. The solution is carried out for two different cases – a weakly ionized and fully ionized medium – for various conditions in the external flow. It is assumed that the cyclotron frequency of the ions is small compared with their collision frequency. The solution is obtained by the method of linearization with respect to a certain parameter. The numerical solutions were obtained on the "Strela" machine in the MGU computation center. Velocity and temperature profiles in the boundary layer are presented for various values of the parameters which define the problem.

Notation

$\mathbf{v}$ – vector (u, v, w) velocity of the medium

p – pressure

p_e – pressure of the electron component

ρ – density

T – temperature

η – coefficient of viscosity

λ – coefficient of heat conductivity in the absence of a magnetic field

c_p – specific heat at constant pressure

c_v – specific heat at constant volume

σ – electric conductivity of the medium in the absence of a magnetic field

$\mathbf{E}$ – vector (E_x, E_y, E_z) of electric field intensity

H_0 – intensity of the uniform magnetic field

$\mathbf{j}$ – vector (j_x, j_y, j_z) of electric current density

ρ_e – electric charge density

ω_1 – cyclotron frequency of electrons

τ_i – "mean free time" of electrons

ρ_0 – characteristic density of the problem

L – characteristic length of the problem

U – characteristic velocity

P – Prandtl number

R – Reynolds number

M – Mach number

Ω – characteristic frequency of the problem

δ – dynamic boundary layer thickness

δ^0 – thermal boundary layer thickness

e – value of the electron charge

k – Boltzmann's constant

c – velocity of light

m_i – masses of electron and ion, respectively, for $i = 1, 2$

n_1 – electron concentration

T_w – temperature of the plate

T_∞ – free stream temperature

$\mathbf{e}_z$ – unit vector along the z-axis

1. The dynamical boundary layer problem. In what follows, the problem always considered is that of the boundary layer on a dielectric plate occupying the half-plane $z = 0$, $x > 0$. The magnetic field is assumed to be uniform and normal to the plate ($\mathbf{H} = H_0\mathbf{e}_z$) in the region $x > 0$ and zero in the region $x < 0$. For $x < 0$ the flow is assumed to be uniform and along the x-axis. It is assumed that $E_y = 0$ and that none of the quantities depend on y. The quantities which characterize the spiral paths of electrons and ions are assumed to satisfy the conditions $\omega_1\tau_1 \equiv \omega\tau \sim 1$, $\omega_2\tau_2 \ll 1$. Under these conditions the viscosity does not depend on the magnetic field. In the following, the viscosity (η), the electrical conductivity in the absence of magnetic field (σ), and $\omega\tau$ are taken to be constant.

The formulation of the dynamical boundary layer problem for these conditions is given in [3], where the problem is solved for one of the possible formulations of conditions in the external flow, for the case of a weakly ionized medium. In the present paper, this problem is solved for other cases.

The problem of the dynamical boundary layer in the formulation under consideration reduces to the system of equations [3]

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} - \frac{1}{R}\frac{\partial^2 u}{\partial z^2} = -\frac{\partial p}{\partial x} + mL(-u + \omega\tau v + \omega\tau E_x^0) + \frac{\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_e}{\partial x}$$

$$u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} - \frac{1}{R}\frac{\partial^2 v}{\partial z^2} = -mL(\omega\tau u + v + E_x^\circ) - \frac{\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_e}{\partial x} \qquad (1.1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

The functions $p = p^*(x)$ and $E_x^\circ = E_x^*(x)$ are determined from the solution of the external problem. In (1.1), as in all that follows, the following dimensionless quantities are used:

$$u = \frac{u}{U}, \quad v = \frac{v}{U}, \quad w = \frac{w}{U}, \quad p = \frac{p}{\rho U^2} \qquad (1.2)$$

$$\mathbf{E} = \frac{c}{UH_0}\mathbf{E}, \quad R = \frac{\rho UL}{\eta}, \quad \mathbf{j} = \frac{L}{cH_0}\mathbf{j}, \quad mL = \frac{\sigma H^2 L}{\rho c^2 U(1+\omega^2\tau^2)}$$

In obtaining equations (1.1), use was made of Ohm's law in the form (with dimensionless quantities)

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) - \frac{\omega\tau}{H}(\mathbf{j}\times\mathbf{H} - c\,\mathrm{grad}\,p_e) \qquad (1.3)$$

When considering a fully ionized medium, it is necessary to put $2p_e = p$ in equations (1.1) and (1.3). For a weakly ionized medium, the terms containing the electron pressure in (1.1) and (1.3) should be dropped.

We shall look for a solution of the system (1.1) of the form

$$u = u_0 + mLu_1, \quad v = mLv_1, \quad w = w_0 + mLw_1, \quad p = p_0 + mLp_1 \text{ etc.} \qquad (1.4)$$

Here, the subscript zero denotes quantities corresponding to the Blasius problem. We introduce the functions $f_0(\xi)$, $f_2(\xi)$, $\Phi(\xi)$ by the formulas

$$u_0 = f_0'(\xi), \quad u_1 = xf_2', \quad w_0 = -\frac{1}{2}\sqrt{\frac{1}{xR}}(f_0 - \xi f_0') \qquad (1.5)$$
$$w_1 = -\frac{1}{2}\sqrt{\frac{x}{R}}(3f_2 - f_2'\xi), \quad v = \omega\tau\, x\Phi(\xi), \quad \xi = z\sqrt{\frac{R}{x}}$$

Taking into account that $\partial p_0/\partial x = 0$, we obtain from (1.1) the following system of equations for determining $f_2(\xi)$ and $\Phi(\xi)$:

$$2f_2''' + f_0 f_2'' - 2f_0' f_2' + 3f_0'' f_2 - 2f_0' - 2\left(\frac{\partial p_1^*}{\partial x} - \omega\tau E_{x0}^*\right) + $$
$$+ 2\frac{\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_{e1}^*}{\partial x} = 0$$
$$2\Phi'' + f_0\Phi' - 2f_0'\Phi - 2f_0' - 2\left(\frac{1}{\omega\tau}E_{x0}^* + \frac{1}{1+\omega^2\tau^2}\frac{\partial p_{e1}^*}{\partial x}\right) = 0 \qquad (1.6)$$

Here, $f_0(\xi)$ is the Blasius function.

To solve equations (1.6), it is necessary to determine $\partial p_1^*/\partial x$ and $E_{x0}^*(x)$ and the boundary conditions for the functions f_2 and Φ at $\xi = \infty$. For this, it is necessary to solve the following equations, which determine the perturbations to the Blasius solution ($u^* = 1$, $v^* = w^* = 0$, $\partial p_0^*/\partial x = 0$) in the external flow

$$\frac{\partial u_1^*}{\partial x} = -\frac{\partial p_1^*}{\partial x} - 1 + \omega\tau E_{x0}^* + \frac{\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_{e1}^*}{\partial x}, \quad \frac{\partial w_1^*}{\partial x} = -\frac{\partial p_1^*}{\partial z}$$
$$\frac{\partial v_1^*}{\partial x} = -\omega\tau - E_{x0}^* - \frac{\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_{e1}^*}{\partial x}, \quad \frac{\partial u_1^*}{\partial x} + \frac{\partial w_1^*}{\partial z} = 0 \qquad (1.7)$$

and also the following equations, which determine the electric field (E_0^*) in the external flow:

$$\operatorname{div} E_0^* = 4\pi\rho_e^\circ = -\omega^2\tau^2\frac{\partial E_{z0}^*}{\partial z} - \frac{\omega\tau}{1+\omega^2\tau^2}\left[(1+\omega^2\tau^2)\frac{\partial p_{1e}^*}{\partial z} + \frac{\partial^2 p_{e1}^*}{\partial x^2}\right]$$
$$\operatorname{rot} E_0^* = 0 \qquad (1.8)$$

Since the plate is taken to be a dielectric, therefore

$$E_{z0}^*(z=0) = 0$$

With equations (1.7) and (1.8), various formulations of the problem are possible.

To solve equations (1.8), what is necessary, in addition to the conditions on the plate $E_{z0}^*(z = 0) = 0$, is to formulate the condition at infinity, which depends on the manner of closure, at infinity, of the currents flowing in the xz-plane. For this condition, use will be made of one of the two following conditions. Either it will be assumed that the potential of the electric field is constant at infinity, which corresponds to full neutralization of the charge carried by the currents in the xz-plane at infinity. In this case the solution of (1.8) has the form $E_0^* = 0$ and the currents in the external flow circulate in the direction of the x-axis. Or it will be assumed that the current density in the xz-plane vanishes at infinity, $\mathbf{j}_{xz\infty}^* = 0$.

In the latter case, it can be shown that it follows from (1.8) that $\mathbf{j}_{xz}^* = 0$ but $\mathbf{E}^* \neq 0$. This results from the fact that the charge which is transported to infinity by the currents in the xz-plane creates an electric field which cancels the field induced by the fluid motion. The magnitude of the electric field can be computed from Ohm's law. We note that for the case being considered ($E_y^* = 0$) the current in the y-direction is always different from zero. If the electric field is given, the external flow has to be determined by solving the system (1.7).

In the following, we shall make use of solutions of (1.7) which correspond either to a uniform flow ($u^* = U + mLu_1^* = U$, $u_1^* = 0$) or to a flow with zero pressure gradient in the x-direction ($\partial p_1^*/\partial x = 0$). The first case corresponds to conditions in which the electromagnetic force in the external flow (it is constant to first approximation) is equal to the constant pressure gradient. The second case corresponds to conditions in which the external flow is braked by the electromagnetic forces. In that case, the velocity in the external flow will be a function of $x (u^*(x) = U + mLu_1^*(x))$, determined by the system (1.7). We note that for the case $\partial p_1^*/\partial x = 0$ it will follow from the equation of continuity that $w \to \infty$ at $z \to \infty$ in the external flow. This indicates that solutions of this type do not exist for the whole flow; they may be considered only as solutions which are valid near the plate and join with a solution away from the plate where $\partial p_1^*/\partial x \neq 0$ (an analogous situation occurred in [4]). In what follows, such a matching will not be considered.

2. Solution of the dynamical boundary layer problem. In solving the boundary layer problem in what follows, four different formulations of the problem in the external flow will be considered, corresponding to the following solutions of equations (1.7) and (1.8):

For a weakly ionized medium

$$j_{x1}^* = 0, \quad u_1^* = 0, \quad E_{x0}^* = -\omega\tau, \quad E_{z0}^* = 0, \quad v_1^* = 0, \quad w_1^* = 0$$

$$\frac{\partial p_1^*}{\partial x} = -(1 + \omega^2\tau^2), \qquad j_{y1}^* = \frac{\rho U^2}{H_0^2}(1 + \omega^2\tau^2), \qquad j_{z1}^* = 0 \qquad (2.1)$$

$$E_{x0}^* = 0, \quad u_1^* = 0, \quad E_{z0}^* = 0, \quad v_1^* = -\omega\tau x, \quad w_1^* = 0, \quad \frac{\partial p_1^*}{\partial x} = -1$$

$$j_{x1}^* = \omega\tau \frac{\rho U^2}{H_0^2}, \quad j_{y1}^* = -\frac{\rho U^2}{H_0^2}, \quad j_{z1}^* = 0 \qquad (2.2)$$

For a fully ionized medium

$$j_{x1}^* = 0, \quad u_1^* = 0, \quad E_{x0}^* = -0.5\,\omega\tau$$

$$E_{z0}^* = 0, \quad v_1^* = 0, \quad w_1^* = 0$$

$$\frac{\partial p_1^*}{\partial x} = -(1 + \omega^2\tau^2)$$

$$j_{y1}^* = -(1 + \omega^2\tau^2)\frac{\rho U^2}{H_0^2}, \quad j_{z1}^* = 0 \qquad (2.3)$$

$$E_{x0}^* = 0, \quad u_1^* = 0, \quad E_{z0}^* = 0$$

$$v_1^* = -0.5\,\omega\tau \frac{1 + \omega^2\tau^2}{1 + 0.5\,\omega^2\tau^2} x, \quad w_1^* = 0$$

$$\frac{\partial p_1^*}{\partial x} = -\frac{1 + \omega^2\tau^2}{1 + 0.5\,\omega^2\tau^2}$$

$$j_{x1}^* = 0.5\,\omega\tau \frac{1 + \omega^2\tau^2}{1 + 0.5\,\omega^2\tau^2} \frac{\rho U^2}{H_0^2} \qquad (2.4)$$

$$j_{y1}^* = -\frac{1 + \omega^2\tau^2}{1 + 0.5\,\omega^2\tau^2} \frac{\rho U^2}{H_0^2}, \quad j_{z1}^* = 0$$

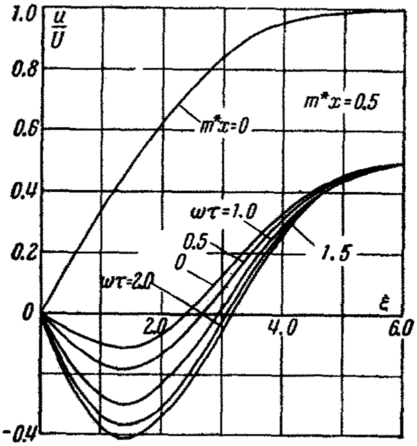

Fig. 1. (2.5)

Formulations which are identical for weakly and fully ionized media are

$$j_{x1}^* = 0, \quad \frac{\partial p_1^*}{\partial x} = 0, \quad E_{x0}^* = -\omega\tau, \quad E_{z0}^* = 0, \quad u_1^* = -(1 + \omega^2\tau^2)\,x$$

$$v_1^* = 0, \quad w_1^* = (1 + \omega^2\tau^2)\,z, \quad j_{y1}^* = -\frac{\rho U^2}{H_0^2}(1 + \omega^2\tau^2), \quad j_{z1}^* = 0 \qquad (2.5)$$

$$E_{x0}^* = 0, \quad \frac{\partial p_1^*}{\partial x} = 0, \quad E_{z0}^* = 0, \quad u_1^* = -x, \quad v_1^* = -\omega\tau x, \quad w_1^* = z$$

$$j_{x1}^* = \omega\tau \frac{\rho U^2}{H_0^2}, \quad j_{y1}^* = -\frac{\rho U^2}{H_0^2}, \; j_{z1}^* = 0 \qquad (2.6)$$

Putting the solutions (2.1) to (2.6) in (1.5) and (1.6), we obtain the ordinary differential equations and boundary conditions for determining the functions $f_2(\xi)$ and $\Phi(\xi)$ for each of the indicated cases. The results, put into the equations

$$u = u_0 + mLu_1 = f_0' + \frac{m^*x}{1 + \omega^2\tau^2} f_2'$$

$$v = mLv_1 = \omega\tau \frac{m^*x}{1 + \omega^2\tau^2} \Phi, \; m^*x = \frac{\sigma H_0^2 x}{\rho U c^2}$$

are shown* in Figs. 1 to 5, which have been obtained for $m^*x = 0.5$ and various values of $\omega\tau$ (naturally, knowing the functions f_2 and Φ, it is possible to obtain the velocity distributions for arbitrary m^*x).

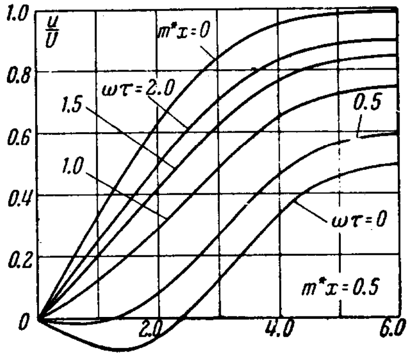

Fig. 2. (2.6)

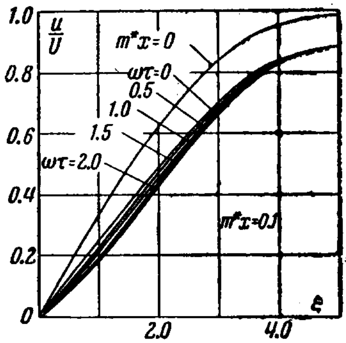

Fig. 3. (2.5)

On all figures, the curve for $m^*x = 0$, corresponding to the Blasius solution, is shown for comparison.

The case (2.2) was considered in [3], but the distribution of the longitudinal velocity u, obtained in [3], will be the same for (2.1) to (2.4), and the distribution of the transverse velocity v will be the same for (2.2) and (2.6).**

Figures 1 and 2 show that for large m^*x (strong magnetohydrodynamic interaction) and for certain values of $\omega\tau$ a separation of the boundary layer occurs in the cross-section $x^* < 0.5\ m^*$. For smaller m^*x and the same $\omega\tau$, separation may not occur. This may be seen, for example, from Fig. 3. In cases (2.1) to (2.4), boundary layer separation does not occur [3], since the flow is with decreasing pressure.

The coefficients of longitudinal and transverse skin friction, C_x and C_y, respectively, may be computed for the various cases from the

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* The authors made calculations of velocity and temperature profiles (cf. below) for $\omega\tau$ equal to 0, 0.5, 1, 1.5, 2 and m^*x equal to 0.1 and 0.5; here, the results of the computations are not given in full. (The numbers under the figures indicate which of the equations (2.1) to (2.6) corresponds to the given curves.)

** Due to the choice of coordinate systems, the velocity v in the present paper and w in [3] are equal in magnitude and opposite in sign.

equations

$$C_x = 2\rho U^{-2}\eta \frac{\partial u}{\partial z}\bigg|_{z=0} = \frac{2}{\sqrt{R_x}}\left[f_0''(0) + \frac{m^* x}{1+\omega^2\tau^2} f_2''(0)\right] \qquad \left(R_x = \frac{\rho U x}{\eta}\right)$$

$$C_y = 2\rho U^{-2}\eta \frac{\partial v}{\partial z}\bigg|_{z=0} = \frac{2}{\sqrt{R_x}} m^* x \frac{\omega\tau}{1+\omega^2\tau^2}\Phi'(0)$$

Here $f_0'' = 0.332$, and the values of $f_2''(0)$ and $\Phi'(0)$ are given in Table 1.*

TABLE 1.

Values of $f_2''(0)$				Values of $\Phi'(0)$			
$\omega\tau$	(2.1) — (2.4)	(2.5)	(2.6)	$\omega\tau$	(2.1), (2.3), (2.5)	(2.4)	(2.2), (2.6)
0	1.139	—0.908	—0.908	0.5	0.871	0.083	—0.547
0.5	1.139	—1.419	—0.908	1	0.871	—0.074	—0.547
1	1.139	—2.955	—0.908	1.5	0.871	—0.213	—0.547
1.5	1.139	—5.513	—0.908	2	0.871	—0.311	—0.547
2	1.139	—9.096	—0.908				

3. Energy equation for the boundary layer in an electrically conducting gas with anisotropic transport properties. Neglecting the spiral paths of the ions ($\omega_2\tau_2 \ll 1$), the energy equation for a fully ionized gas may be written in the form [1]

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + \mathbf{j}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) + \eta\left[2\sum_i\left(\frac{\partial v_i}{\partial x_i}\right)^2 - (\operatorname{div}\mathbf{v})^2\right] +$$
$$+ \eta\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2\right] + \frac{1}{3}\eta(\operatorname{div}\mathbf{v})^2 +$$
$$+ \operatorname{div}(\lambda\varkappa\nabla T) + \frac{\partial}{\partial z}\lambda(1-\varkappa)\frac{\partial T}{\partial z} + \operatorname{div}\lambda\iota'\mathbf{j} + \frac{\partial}{\partial z}\lambda(\iota-\iota')j_z -$$
$$- \left(\frac{\partial}{\partial x}\lambda\omega\tau\varkappa'\frac{\partial T}{\partial y} - \frac{\partial}{\partial y}\lambda\omega\tau\varkappa'\frac{\partial T}{\partial x}\right) -$$
$$- \left(\frac{\partial}{\partial x}\lambda\omega\tau\iota'' j_y - \frac{\partial}{\partial y}\lambda\omega\tau\iota'' j_x\right) \qquad (3.1)$$

* The values of $f_2''(0)$ given here are different from the corresponding values given in [4] and taken over by [3] from [4]. Also, we take the opportunity to note that the value of $\Phi'(0)$ for case (2.2) given in [3] is incorrect.

Here

$$\varkappa = (1{,}47\omega^2\tau^2 + 3.77)\Delta, \qquad \varkappa' = (0.791\omega^2\tau^2 + 6.86)\,\Delta$$

$$\iota = 2.03,\ \frac{TH_0}{\omega\tau pc}, \qquad \iota' = (1.58\omega^4\tau^4 + 26.6\omega^2\tau^2 + 7.66)\frac{TH_0\Delta}{\omega\tau pc}$$

$$\iota'' = (0.949\omega^2\tau^2 + 1.93)\frac{TH_0\Delta}{\omega\tau pc}, \qquad \frac{1}{\Delta} = \omega^4\tau^4 + 14.79\omega^2\tau^2 + 3.77$$

Equation (3.1) is written for the case of the magnetic field directed along the z-axis.

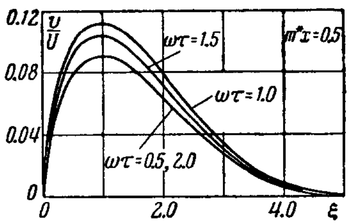

Fig. 4.(2.1), (2.3), (2.5)

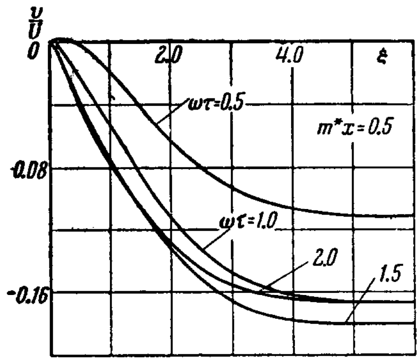

Fig. 5. ((2.4)

We note that in a fully ionized gas the viscosity is determined by the ions and the heat conductivity (for the case $\omega_2\tau_2 \ll 1$) by the electrons. In equation (3.1), besides the usual terms, connected with the addition of heat due to heat conduction, viscous and joule dissipation, and the work of pressure forces, there are terms connected with the addition of heat due to electron diffusion (Thomson effect)

$$\left[\operatorname{div} \lambda\iota'\mathbf{j} + \frac{\partial}{\partial z}\lambda\,(\iota - \iota')\,j_z\right]$$

and due to the spiral electron paths (Ettingshause, Leduc-Righi effects [1])

$$\frac{\partial}{\partial x}\lambda\omega\tau\iota'' j_y - \frac{\partial}{\partial y}\lambda\omega\tau\iota'' j_x + \frac{\partial}{\partial x}\lambda\omega\tau\varkappa'\frac{\partial T}{\partial y} - \frac{\partial}{\partial y}\lambda\omega\tau\varkappa'\frac{\partial T}{\partial x}$$

In order to obtain the boundary layer equations from equations (3.1), it is necessary to make the usual boundary layer estimates [2,5] and keep in the equations only the main terms. Doing this, we obtain from (3.1), in the formulation of Article 1 ($\mathbf{H} = H_0\mathbf{e}_z$, $E_y = 0$, $\partial/\partial y = 0$, $H_0 = \text{const}$), the following energy equation for the boundary layer on a flat plate:

$$\rho c_p\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = u\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z} + \eta\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] -$$

$$-\frac{2}{3}\eta\,(\mathrm{div}\,\mathbf{v})^2 + j_x{}^\circ\Big(E_x{}^\circ + \frac{H_0}{c}v\Big) - \frac{H_0}{c}j_y{}^\circ u + \frac{\partial}{\partial z}\lambda\iota j_z{}^1 + \\ + \frac{\partial}{\partial x}(\lambda\iota' j_x{}^\circ) - \frac{\partial}{\partial x}(\omega\tau\lambda\iota'' j_y{}^\circ) \qquad (3.2)$$

Here, $j_x{}^\circ$, $j_y{}^\circ$, $j_z{}^\circ$, $E_x{}^\circ$ and $E_y{}^\circ$ are the components of current density and electric field strength [2], of order unity (with respect to δ), and $j_z{}^1$ is the current density in the z-direction, of order δ.

For the boundary layer on a dielectric plate [3] we have $j_z{}^\circ = 0$. In what follows, we shall use, in addition to (1.2), the following dimensionless quantities:

$$\rho = \frac{\rho}{\rho_0}, \quad T = \frac{c_pT}{U^2}, \quad P = \frac{\eta c_p}{\lambda}, \quad B = \frac{\sigma k H_0}{ce\rho_0 c_p} \qquad (3.3)$$

Here, ρ_0 is a characteristic density, e is the charge on the electron, and the other symbols are well known.

Using Ohm's law in the form (1.3) and the dimensionless quantities (1.2) and (3.3), we obtain equations (3.2) in the form

$$\frac{\rho}{\rho_0}\Big(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\Big) = u\frac{\partial p}{\partial x} + \frac{1}{PR}\frac{\partial^2 T}{\partial z^2} + \frac{1}{R}\Big[\Big(\frac{\partial u}{\partial z}\Big)^2 + \Big(\frac{\partial v}{\partial z}\Big)^2\Big] - \\ \frac{2}{3}\eta\,(\mathrm{div}\,v)^2 + mL\Big[(E_x{}^\circ + v)\Big(E_x{}^\circ + v + \omega\tau u + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big) - \\ - u\Big(\omega\tau E_x{}^\circ + \omega\tau v - u + \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big)\Big] + 3.21\,B\frac{\partial}{\partial z}(Tj_z{}^1) + \\ + B\frac{\partial}{\partial x}\Big\{T\Big[\frac{\alpha}{1+\omega^2\tau^2}\Big(E_x{}^\circ + v + \omega\tau u + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big) - \\ - \frac{\omega\tau\beta}{1+\omega^2\tau^2}\Big(\omega\tau E_x{}^\circ - u + \omega\tau v + \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big)\Big]\Big\} \qquad (3.4)$$

Here

$$\alpha = 1.58\,\frac{1.58\,\omega^4\tau^4 + 26.6\,\omega^2\tau^2 + 7.66}{\omega^4\tau^4 + 14.79\,\omega^2\tau^2 + 3.77}, \qquad \beta = 1.58\,\frac{0.95\,\omega^2\tau^2 + 1.93}{\omega^4\tau^4 + 14.79\,\omega^2\tau^2 + 3.77}$$

In obtaining this equation and in what follows, it is assumed for simplicity that the specific heat, the transport coefficients, and $\omega\tau$ are constants.

It is easy to see that, in the case being considered ($j_x{}^\circ \sim j_y{}^\circ \sim 1$, $j_z \sim \delta$), the terms connected with the Thomson and Ettingshausen effects are of the same order, while their relation to terms on the left side of equation (3.4) are determined by the parameter B. If the terms making up B are represented by formulas from kinetic theory, we obtain

$$B = \frac{\sigma k H_0}{c \rho_0 e c_p} = \frac{n_1 e^2 \tau_1}{m_1} \frac{k H_0}{c e \rho_0} \frac{m_2}{5k} = \frac{\rho}{\rho_0} \frac{\omega\tau}{5} \tag{3.5}$$

Here, c_p is expressed by means of the usual formulas for a mixture composed of electrons and singly-ionized ions, neglecting the specific heat connected with the potential energy of interaction of the particles

$$c_p = \frac{m_1}{m_2} c_{p1} + c_{p2} = \frac{m_1}{m_2} \frac{5}{2} \frac{k}{m_1} + \frac{5}{2} \frac{k}{m_2} = 5 \frac{k}{m_2}$$

Thus, the parameter B for a fully ionized gas is related to $\omega\tau$ and is not an independent parameter of the problem. For $\omega\tau \sim 1$ the parameter $B \sim 1$, and, therefore, terms connected with the Thomson and Ettingshausen effects are comparable with the convective terms in equations (3.4). In what follows, the energy equation for a fully ionized gas will be used in the form (3.4).

The quantity j_z^1 is determined by the relation

$$j_z^1 = (1 + \omega^2\tau^2)\, mL \frac{\rho U^2}{H_0^2} \left(E_z^1 + \frac{0.5\,\omega\tau}{1 + \omega^2\tau^2} \frac{1}{mL} \frac{\partial p}{\partial z} \right) \tag{3.6}$$

To determine E_z^1 we make use of the continuity equation for current density, the generalized Ohm's law in the form (1.3), as well as boundary layer estimates in Maxwell's equations [2,3], and obtain

$$\frac{\partial}{\partial z} \left(E_z^1 + \frac{0.5\,\omega\tau}{1 + \omega^2\tau^2} \frac{1}{mL} \frac{\partial p}{\partial z} \right) =$$
$$= \frac{1}{1 + \omega^2\tau^2} \left(\omega\tau \frac{\partial w}{\partial z} - \frac{\partial v}{\partial x} - \frac{0.5\,\omega\tau}{1 + \omega^2\tau^2} \frac{1}{mL} \frac{\partial^2 p}{\partial x^2} - \frac{\partial E_x^\circ}{\partial x} \right) \tag{3.7}$$

It should be noted that, for a fully ionized gas, the Prandtl number is a small quantity. In fact

$$P = \frac{\eta c_p}{\lambda} = 5 \frac{k}{m_2} 0.96\, nkT\tau_2 \frac{m_1}{1.58\, pk\tau_1} \sim \frac{nkT}{p} \frac{m_1}{m_2} \frac{\tau_2}{\tau_1} \sim \sqrt{\frac{m_1}{m_2}} \ll 1 \tag{3.8}$$

In addition, we note that, in a fully ionized gas, the following relation exists between the Reynolds number and other parameters defining the problem:

$$R = \frac{\rho U L}{\eta} \sim \frac{1}{\omega\tau} \frac{\omega_2}{\Omega} \frac{\gamma U^2}{(\gamma - 1) c_p T} \frac{1}{P} \sim \frac{1}{\omega\tau} \frac{\omega_2}{\Omega} \frac{1}{P} \gamma M^2 \sim \frac{m^* L}{\omega^2\tau^2} \frac{1}{P} \gamma M^2$$
$$\left(\gamma = \frac{c_p}{c_v}, \quad m^* L = \frac{\sigma H_0^2 L}{\rho c^2 U} \right) \tag{3.9}$$

Here, M is the Mach number, Ω is a characteristic frequency of the problem. The relation (3.9) shows that in a fully ionized gas

$$m^*L = \omega\tau\omega_2 / \Omega \sim 1 \quad \text{for} \quad \Omega / \omega_2 \sim \omega\tau \sim 1$$

(in this case, Ohm's law [6] is of the form (1.3)).

In this case the boundary layer equations are valid only for large free stream velocities ($M^2 \gg 1$), since, besides satisfying the inequality

$$(\delta^2) \sim \frac{1}{R} \ll 1$$

the condition of small thermal boundary layer thickness δ° must also be satisfied [5]

$$(\delta^\circ)^2 = \left(\frac{\delta^\circ}{L}\right)^2 \sim \frac{1}{RP} \ll 1$$

If $\omega\tau$ is made smaller, with H_0 and other parameters constant, then it follows from (3.9) that the Reynolds number increases, which is connected with the decrease of viscosity, inasmuch as

$$\tau_2 \sim \sqrt{\frac{m_2}{m_1}}\,\tau_1, \qquad R \sim (\gamma - 1)\, M^2 \frac{L}{U\tau_2} \tag{3.10}$$

For increasing $\omega\tau$, the Reynolds number decreases. Thus, for a fully ionized gas, the Reynolds number will not be independent of the parameters $\omega\tau$ and Ω/ω_2, which determine the electromagnetic effects on the flow. This connection is determined by the relations (3.9). For independently given M, $\omega\tau$, Ω/ω_2 (M, $\omega\tau$, M^*L), the Reynolds number becomes a determined quantity, and therefore, in such cases, it is always necessary to verify the applicability of the boundary layer model.

If the gas is partially ionized, then, as follows from [7], the coefficients in the Thomson and Ettingshausen effects in equation (3.2) are, roughly speaking, proportional to the degree of ionization. Therefore, in considering the boundary layer in a weakly ionized gas, the corresponding terms in the energy equation may be neglected, while the generalized Ohm's law will have the form (1.3), but without the term containing the electron gas pressure gradient. In addition, in a weakly ionized gas, the heat conductivity and the viscosity are determined by the neutral particles, and therefore, the Prandtl number is of order unity.

Thus, in a weakly ionized gas, the energy equation for the boundary layer for the case considered ($j_z{}^\circ = 0$) can be written in the form (cf. (3.4))

$$\frac{\rho}{\rho_0}\left(u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=u\frac{\partial p}{\partial x}+\frac{1}{PR}\frac{\partial^2 T}{\partial z^2}+\frac{1}{R}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]- \qquad (3.11)$$
$$-\frac{2}{3}\eta\,(\mathrm{div}\ v)^2+mL\,[(E_x{}^\circ+v)\,(E_y{}^\circ+v+\omega\tau u)-u\,(\omega\tau E_x{}^\circ+\omega\tau v-u)]$$

The parameters P, R, mL and $\omega\tau$ for a weakly ionized gas are independent, since P and R are determined by the neutral gas and do not depend on mL and $\omega\tau$, which are connected with the charged components. Equation (3.11) differs from the corresponding equation in isotropic megnetohydrodynamics [4] only in a change in the term connected with joule heat.

4. Thermal boundary layer on a flat plate in a flow of weakly ionized incompressible gas with anisotropic electrical conductivity. The energy equation for a weakly ionized incompressible medium will be written in the form

$$u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\frac{1}{RP}\frac{\partial^2 T}{\partial z^2}+\frac{1}{R}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]+mL\,[(E_x{}^\circ+v)\times \qquad (4.1)$$
$$\times\,(E_x{}^\circ+v+\omega\tau u)-u\,(\omega\tau E_x{}^\circ+\omega\tau v-u)] \qquad \left(T=\frac{c_v T}{U^2},\ P=\frac{\eta c_v}{\lambda}\right)$$

Here, all quantities are dimensionless; c_v is the specific heat at constant volume.

Analogously to the case in Section 1, we shall look for solutions of equations (4.1) in the form

$$T=T_0+mLT_1 \qquad (4.2)$$

Putting relations (1.4) and (4.2) in (4.1) we obtain for T_0 the equation

$$u_0\frac{\partial T_0}{\partial x}+w_0\frac{\partial T_0}{\partial z}=\frac{1}{R}\left[\frac{\partial^2 T_0}{\partial z^2}+\left(\frac{\partial u_0}{\partial z}\right)^2\right] \qquad (4.3)$$

Here, and in all the computed examples of the case under consideration, we take $P = 1$. The solution of equation (4.3) corresponds to the solution for the distribution of temperature in an incompressible fluid in an ordinary boundary layer ($H_0 = 0$). With the boundary conditions

$$T_0=T_w \quad \text{for } z=0, \qquad T_0=T_\infty=T_w=\text{const} \quad \text{for } z=\infty$$

the solution of (4.3) has the form

$$T_0 = T_w + \theta\ (\xi) = T_w - \tfrac{1}{2}\,[1 - f_0'\,(\xi)] + \tfrac{1}{2}\,[1 - f_0'^2\,(\xi)] \qquad (4.4)$$
$$\xi = z\,\sqrt{R/x}$$

Here, $f_0(\xi)$ is the Blasius function. For T_1 we obtain the equation

$$u_0 \frac{\partial T_1}{\partial x} + w_0 \frac{\partial T_1}{\partial z} - \frac{1}{R}\frac{\partial^2 T_1}{\partial z^2} = -u_1 \frac{\partial T_0}{\partial x} - w_1 \frac{\partial T_0}{\partial z} + \frac{2}{R}\frac{\partial u_0}{\partial z}\frac{\partial u_1}{\partial z} +$$
$$+ E_{x0}^\circ (E_{x0}^\circ + \omega\tau u_0) - u_0 (\omega\tau E_{x0}^\circ - u_0) \qquad (4.5)$$

Here, u_0, w_0 are determined from the Blasius solution, T_0 is determined from equation (4.4), u_1, w_1 and E_{x0}° are determined from the solution of the dynamical boundary layer and the external flow in Section 2. We shall seek a solution for T_1 in the form

$$T_1 = x\Psi\,(\xi) \qquad (4.6)$$

Then, equation (4.5) can be transformed to the form

$$f_0'\Psi - \tfrac{1}{2} f_0 \Psi' - \Psi'' = -\tfrac{3}{2} f_2 f_0'' \left(f_0' - \tfrac{1}{2}\right) + 2f_0'' f_2'' + f_0'^2 + (E_{x0}^\circ)^2 \qquad (4.7)$$

Since the temperature of the plate is assumed to be given, the boundary condition for the function $\Psi(\xi)$ will be

$$\Psi\,(0) = 0 \qquad \text{for } \xi = 0 \qquad (4.8)$$

In the external flow (for $\xi = \infty$), there are electrical currents. Due to joule heating, these currents heat the fluid, and therefore the temperature of the external flow will not be constant, if terms of order mL are taken into account. To determine the function T_1 in the external flow, it is necessary to solve the equation

$$\frac{\partial T_1^*}{\partial x} = 1 + (E_{x0}^\circ)^2 \qquad (4.9)$$

which is easily obtained from (4.1) by putting $w = 0$ and $R = \infty$ and keeping only terms of order mL. The solution of (4.9) has the form ($T_1 = 0$ for $x = 0$)

$$T_1^* = (1 + E_{x0}^{\circ 2})\,x$$

Using this solution, we obtain the following boundary condition:

$$\Psi\,(\infty) = 1 + (F_{x0}^\circ)^2 \quad \text{for } \xi = \infty \qquad (4.10)$$

Putting into (4.7) and (4.10) the values $E_{x0}{}^{*} = E_{x0}{}^{\circ}$ in the external flow, for cases (2.1) to (2.6), we obtain for these cases the equations which determine the function Ψ. These were solved numerically.

Figures 6 and 7 show the behavior of the function

$$\Theta = T_0 + \frac{m^*x}{1+\omega^2\tau^2}\Psi(\xi) \qquad \left(\Theta = \frac{c_v(T-T_w)}{U^2}\right) \tag{4.11}$$

which gives the dimensionless temperature difference between a point in the flow and the wall. The curves are for $m^*x = 0.5$ and various values of $\omega\tau$.

The heat transfer coefficient, to terms of order mL, for a weakly ionized medium will have the form

$$h = \frac{c_v q}{U^2} = \frac{c_v\lambda}{U^2}\frac{dT}{dz}\bigg|_{z=0} = \tag{4.12}$$

$$= \frac{\rho U c_v}{\sqrt{R_x}}\left[0.166 + \frac{m^*x}{1+\omega^2\tau^2}\Psi'(0)\right]$$

TABLE 2.

Values of $\Psi'(0)$

$\omega\tau$	(2.1), (2.5)	(2.2), (2.6)
0	—0.176	—0.176
0.5	—0.040	—0.176
1	0.389	—0,176
1.5	1.355	—0.176
2	2.104	—0.176

The values of $\Psi'(0)$ for various cases may be found in Table 2.

Figures 6 and 7 show that the boundary layer heating in cases (2.2) and (2.6) decreases with increasing $\omega\tau$, which is connected with a decrease of the effective conductivity. In cases (2.1) and (2.5), the boundary layer heating increases with increasing $\omega\tau$. For $\omega\tau \to 0$, the temperature profiles become identical for all cases.

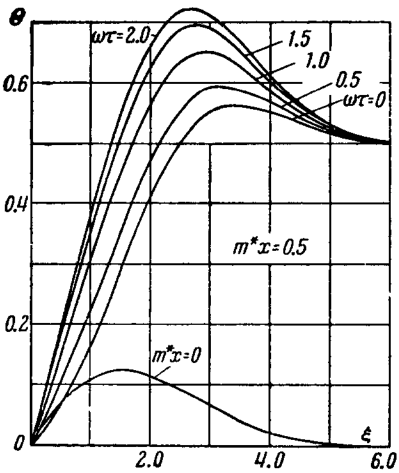

Fig. 6. (2.1),(2.5)

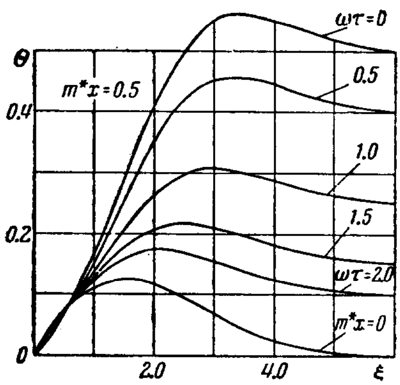

Fig. 7. (2.2),(2.6)

5. Thermal boundary layer on a flat plate in a flow of fully ionized incompressible gas with anisotropic transport properties. Using (3.4) and (3.6), we have the energy equation

$$u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\frac{1}{PR}\frac{\partial^2 T}{\partial z^2}+\frac{1}{R}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]+mL\Big[(E_x{}^\circ+v)\Big(E_x{}^\circ+v+\omega\tau u+$$
$$+\frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big)-u\left(\omega\tau E_x{}^\circ+\omega\tau v-u+\frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\right)\Big]+$$
$$+3.21\,B\frac{\partial}{\partial z}\left[T\left(E_z{}^1+\frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial z}\right)\right]+B\frac{\partial}{\partial x}\Big\{T\Big[\frac{\alpha}{1+\omega^2\tau^2}\Big(E_x{}^\circ+v+\omega\tau u+$$
$$+\frac{0,5\,\omega\tau}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\Big)-\frac{\omega\tau\beta}{1+\omega^2\tau^2}\left(\omega\tau E_x{}^\circ-u+\omega\tau v+\frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{1}{mL}\frac{\partial p}{\partial x}\right)\Big]\Big\} \qquad (5.1)$$

$$T=c_vT/U^2,\quad P=\eta c_v/\lambda,\quad B=\sigma kH_0/cepc_v=\omega\tau/3$$

We shall look for a solution of (5.1) of the form of (1.4) and (4.2). Then, in the zero-th approximation with respect to mL, we obtain

$$u_0\frac{\partial T_0}{\partial x}+w_0\frac{\partial T_0}{\partial z}-\frac{1}{RP}\frac{\partial^2 T_0}{\partial z^2}-\frac{1}{R}\left(\frac{\partial u_0}{\partial z}\right)^2=\frac{\omega\tau}{3(1+\omega^2\tau^2)}\Big\{3.21\frac{\partial T_0 w_0}{\partial z}\bar{\omega\tau}+$$
$$+\frac{\partial}{\partial x}T_0\Big[\alpha\Big(E_{x0}{}^\circ+\omega\tau u_0+\frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\Big)-\beta\omega\tau\left(\omega\tau E_{x_0}{}^\circ-u_0+\frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\right)\Big]\Big\} \qquad (5.2)$$

In obtaining (5.2) and in the problems that follow in this paper, use was made of equation (3.7), taking into account the smallness of the quantity mL.

The maximum value of the parameter $\omega\tau/3(1+\omega^2\tau^2)$, which appears in front of the expression on the right-hand side of (5.2) occurs at $\omega\tau=1$ and is equal to 1/6. For $\omega\tau\to 0$ or $\omega\tau\to\infty$ this parameter approaches zero. In view of this, to simplify the problem we shall look for a solution of (5.2) of the form

$$T_0=T_w+\theta(\xi)+\frac{\omega\tau}{3(1+\omega^2\tau^2)}F(\xi),\quad \xi=z\sqrt{\frac{R}{x}} \qquad (5.3)$$

Considering the parameter $\omega\tau/3(1+\omega^2\tau^2)$ to be small, we obtain from (5.2) the following equations for $\theta(\xi)$ and $F(\xi)$:

$$\theta''+\frac{P}{2}f_0\theta'+Pf_0''^2=0 \qquad (5.4)$$

$$\frac{1}{P}F''+\frac{1}{2}f_0F'=3.21\frac{\omega\tau}{2}\left[(f_0-\xi f_0')\theta'-\xi f_0''(T_w+\theta)\right]+$$
$$+\frac{1}{2}\left[\alpha\left(E_{x0}{}^\circ+\frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\right)-\omega\tau\beta\left(\omega\tau E_{x0}{}^0+\frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\right)\right]\xi\theta'+$$
$$+\omega\tau(\alpha+\beta)\frac{\xi}{2}\left[f_0'\theta'+(T_w+\theta)f_0''\right] \qquad (5.5)$$

Of course, it would be possible to look for a solution of T_0 directly from equation (5.2), without making use of the smallness of $\omega\tau/3(1 + \omega^2\tau^2)$, especially since equation (5.2) has a similarity solution in terms of the Blasius parameter. However, the course adopted here, to seek a solution of the form (5.3), turns out to be more convenient.

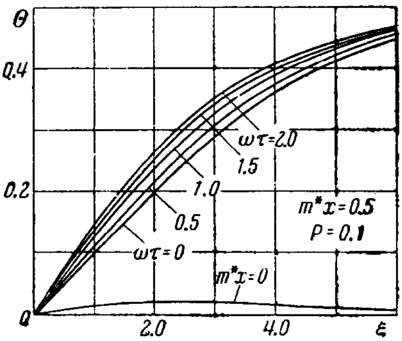

Fig. 8. (2.3)

It is easy to show that equation (5.2), when written for the external flow, has the solution T_0 = const. Therefore, considering that $T_w = T_\infty$, the solutions of equations (5.4) and (5.5) should have the boundary conditions

$$\theta\,(0) = 0, \quad \theta\,(\infty) = 0 \tag{5.6}$$

$$F\,(0) = 0, \quad F\,(\infty) = 0 \tag{5.7}$$

From equation (5.1), neglecting all products of the small quantities mL and $\omega\tau/3(1 + \omega^2\tau^2)$, we obtain for T_1 the equation

$$u_0\frac{\partial T_1}{\partial x} + w_0\frac{\partial T_1}{\partial z} - \frac{1}{RP}\frac{\partial^2 T_1}{\partial z^2} = -u_1\frac{\partial\theta}{\partial x} - w_1\frac{\partial\theta}{\partial x} + \frac{2}{R}\frac{\partial u_0}{\partial z}\frac{\partial u_1}{\partial z} + E_{x_0}{}^{\circ}\left(E_{x_0}{}^{\circ} + \omega\tau u_0 + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\right) - u_0\left(\omega\tau E_{x_0}{}^{\circ} - u_0 + \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}\right) \tag{5.8}$$

We shall look for a solution of (5.8) in the form

$$T_1 = x\Psi_1(\xi) \tag{5.9}$$

To determine $\Psi_1(\xi)$ we then obtain the equation

$$f_0{}'\Psi_1 - \frac{1}{2}f_0\Psi_1{}' - \frac{1}{P}\Psi_1{}'' = \frac{3}{2}f_2\theta' + 2f_0{}''f_2{}'' + f_0{}'^2 + (E_{x_0}{}^{\circ})^2 + E_{x_0}{}^{\circ}\frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x} - f_0{}'\frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x} \tag{5.10}$$

To set up the boundary conditions for equation (5.10), the equation for the external flow analogous to (5.8) has to be solved:

$$\frac{\partial T_1{}^*}{\partial x} = 1 - \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x} + (E_{x_0}{}^{\circ})^2 + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}E_{x_0}{}^{\circ}$$

The solution of this equation has the form

$$T_1{}^* = \left[1 - \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x} + (E_{x_0}{}^{\circ})^2 + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1{}^*}{\partial x}E_{x_0}{}^{\circ}\right]x$$

From this, the boundary conditions for equation (5.10) follow:

$$\Psi_1(0) = 0, \quad \Psi_1(\infty) = \left[1 - \frac{0.5\,\omega^2\tau^2}{1+\omega^2\tau^2}\frac{\partial p_1^*}{\partial x} + (E_{x_0}{}^\circ)^2 + \frac{0.5\,\omega\tau}{1+\omega^2\tau^2}\frac{\partial p_1^*}{\partial x}E_{x_0}{}^\circ\right] \tag{5.11}$$

Specific computations of problems (5.4), (5.6); (5.5), (5.7); (5.10), (5.11) were carried through for various values of $\omega\tau$, m^*x, and for values of the Prandtl number $P = 0.1$ and $P = 0.01$. For the integrations, it was assumed that $T_w \equiv \dot{c}_v T_w/U^2 = 0.1$.

Figures 8 to 11 show, for illustration, the behavior of the function

$$\Theta = \theta(\xi) + \frac{\omega\tau}{3(1+\omega^2\tau^2)}F(\xi) + \frac{m^*x}{1+\omega^2\tau^2}\Psi_1(\xi) \tag{5.12}$$

which corresponds to the dimensionless temperature difference between a point in the flow and the wall for $m^*x = 0.5$ and $P = 0.1$ for various cases (2.3) to (2.6).

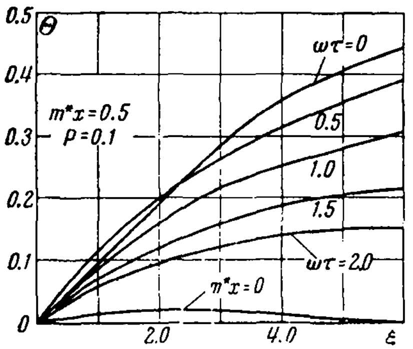

Fig. 9. (2.4)

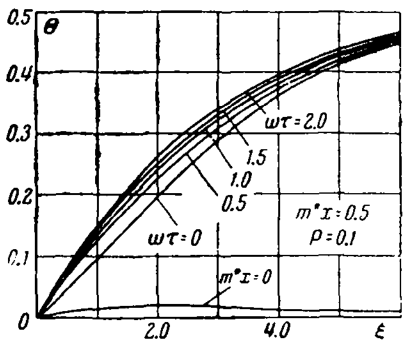

Fig. 10. (2.5)

Figures 8 to 11 show that a determining factor in the temperature distribution is the heating of the fluid in the outer flow. In regard to this, it is clear from physical considerations that the greater the heat conductivity (smaller Prandtl number) the closer the temperature profile must approach the linear one. The computations for $P = 0.01$ confirm this conclusion. For those cases in which the heating of the fluid in the outer flow does not depend on $\omega\tau$, the temperature distribution curves for different $\omega\tau$ practically merge into a single curve, nearly a straight line. For those cases in which the heating in the outer flow depends on $\omega\tau$, the temperature distribution is different for different $\omega\tau$, but for each $\omega\tau$ it is nearly linear (cf., for example, Fig. 12).

The coefficient of heat transfer, to terms of order mL, for a fully ionized medium will have the form

$$h = \frac{c_v q}{U^2} = \frac{c_v \lambda}{U^2} \frac{\partial T}{\partial z}\bigg|_{z=0} = \frac{\rho U c_v}{\sqrt{R_x}} \left[\theta'(0) + \frac{m^* x}{1+\omega^2\tau^2} \Psi_1'(0) + \frac{\omega\tau}{3(1+\omega^2\tau^2)} F'(0) \right] \quad (5.13)$$

The values of $\Psi'(0)$ and $F'(0)$ for different cases are given in Table 3 ($\theta'(0) = 0.021$ for $P = 0.1$, and $\theta'(0) = 0.002$ for $P = 0.01$).

TABLE 3.

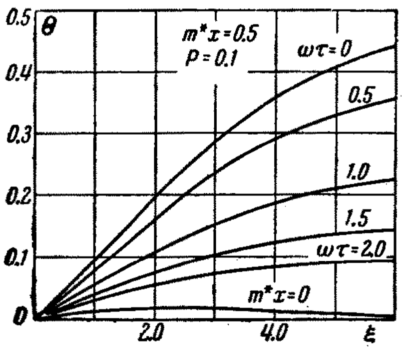

Fig. 11. (2.6)

Values of $F'(0)$

$\omega\tau$	P	(2.3), (2.5)	(2.4)	(2.6)
0.5	0.1	—0.0007	0.0001	0.0009
	0.01	0	0	0
1.0	0.1	0.0006	0.0026	0.0036
	0.01	0	0	0
1.5	0.1	0.0022	0.0055	0.0066
	0.01	0.0002	0.0002	0.0002
2.0	0.1	0.0042	0.0087	0.0096
	0.01	0.0004	0.0004	0.0004

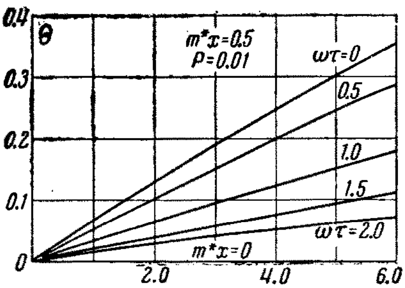

Fig. 12. (2.6)

Values of $\Psi_1'(0)$

$\omega\tau$	P	(2.3)	(2.4)	(2.5)	(2.6)
0	0.1	0.120	0.120	0.120	0.120
	0.01	0.130	0.130	0.130	0.130
0.5	0.1	0.132	0.276	0.195	0.120
	0.01	0.166	0.147	0.154	0.130
1	0.1	0.541	0.358	0.391	0.120
	0.01	0.271	0.180	0.256	0.130
1.5	0.1	0.922	0.430	0.719	0.120
	0.01	0.447	0.209	0.427	0.130
2	0.1	1.457	0.480	1.178	0.120
	0.01	0.692	0.229	0.665	0.130

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